Thinking Creatively
about Teaching Geometry

A Teacher’s Guide
to Using GeoGebra
in Elementary School

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1. Imagining Teaching Geometry Dynamically

Why do we teach geometry in elementary school? More than just the study of shapes, lines, and angles, geometry is a playground for rational thinking, one that can engage any thinker at any level. For children, it is a field where they can play, explore, and develop. Emerging mathematical thinking, for some, is a first step on a powerful journey towards deep understanding in a life filled with ideas and innovation.

Yet, there are others who never gain access to this playground, children whose natural curiosity does not develop into a love of mathematics nor into robust spatial and numeric literacy. The challenge for elementary teachers is to unlock these gates and to provide
meaningful and engaging opportunities for mathematical play, exploration, and development for all students. If we think of our role as teachers as being designers of educational experiences, part of our challenge is to create the needed scaffolded activities within our elementary classrooms to nurture students’ curiosity and to help them figure out how to apply a rational approach towards understanding the world around them.

This tutorial highlights the ways that dynamic geometry software offers exciting new possibilities for exploring geometry with elementary-aged children, as well as strategies for meaningful incorporation of these new ideas and tools into your regular classroom practice. Moving beyond the GeoGebra QuickStart Geometry Tutorial that will get you started as a user, this tutorial explores the needed pedagogical strategies and perspectives that will help you to use GeoGebra more effectively as an elementary school teacher.

I focus on GeoGebra because it is free, and therefore easily accessible, and also because of the potential I see in the creative-commons, sharing community which has quickly grown up around it. GeoGebra is not, however, the first dynamic geometry software and it is not necessarily the best. The first, according to Wikipedia, was Geometric Supposer back in the early 1980’s, soon followed by Cabri Géomètre in France in 1986 and Geometer’s Sketchpad in the US in 1991. Wikipedia now lists 37 different types of dynamic geometry software with 2D capabilities, and 12, including GeoGebra 5.0, that model in three dimensions.

GeoGebra was created by Markus Hohenwarter, a master’s student at the time in Austria, and released in 2002. It combined aspects of existing geometry software with existing algebra software. GeoGebra’s most radical departure is a business and development model rooted in an international, open-source community. Organizationally, GeoGebra is actually a hybrid model with an international network of nonprofit entities and a for-profit section based in Austria that offers non-commercial licenses for free and charges for commercial-use licenses.
GeoGebra is available in over 60 languages, translated by teams of volunteers, and is multi-platform, available either through a web browser or as a stand-alone, downloadable application that works across most devices. Thoroughly 2.0, it promotes a culture of sharing and interaction through forums and community blogs, and since 2011, through a file sharing hub called GeoGebraTube.

As a sharing community, GeoGebraTube has powerful -- though as yet largely underdeveloped -- potential. With its 370,000 shared resources (as of March, 2016), an elementary teacher might imagine that GeoGebraTube would offer a wealth of scaffolded materials for use in the classroom. The reality is better described as an unintelligible sea of half-formed dynamic ideas. I fear that many curious teachers download GeoGebra, or access it through their web-browser, and explore for an hour or two before getting frustrated and lost.

A certain amount of frustration, of course, can be a constructive part of learning, but in this case, primarily because of a lack of the particular kinds of supports that an elementary teacher needs, this frustration more likely leads to discouragement and abandonment. Even if a teacher manages to find their way to the QuickStart Geometry Tutorial on their own, without further support relating to how and why to use GeoGebra in the classroom, the way ahead will undoubtedly be daunting. These days, there is simply too much else available on the Internet to bother wasting one’s time on a resource that is not user-friendly.

With the right introduction and support, however, the world of dynamic geometry can energize a mathematics classroom and open up an exciting world for teachers and for students. From middle school onwards, digital tools for algebra, spreadsheets, simulations, and geometry have become the norm. At the elementary level, in contrast, digital tools for mathematics are often limited to apps that simply drill skills. This is at odds with an overview of the existing research (Chan & Leung, 2014) that indicates that dynamic geometry software used at the
elementary level inspires more dramatic gains than when used in middle and upper school. Perhaps, this finding is influenced by the fact that these studies at the elementary level are often led by innovative teachers modeling new pedagogical approaches (Bu & Schoen, 2011; Clark-Wilson, Robutti & Sinclair, 2014; Cho, 2015; Ng & Sinclair, 2015; Sinclair & Bruce, 2015). The current challenge, as I see it, is to make these effective techniques for teaching dynamic geometry more accessible to more elementary teachers.

2. Thinking Creatively

As a structural framework, this tutorial relies on the insights of Mitchel Resnick (2007) into creative thinking, which he describes as an iterative process of imagining, creating, playing, sharing, and reflecting. The first step, before you jump in and start trying to create, is to wet your imagination. Let’s take a few minutes to explore a few of the beautiful and intriguing constructions of Malin C. (Christersson, 2015) through which you can catch a glimpse of what it is possible to construct with GeoGebra. Do not be frightened; some of these are pitched at upper grade levels. Just imagine for a moment the many places this dynamic journey might lead.
3. The GeoGebra QuickStart Tutorial: Getting Started

   This chapter offers a crash course to help you get started with GeoGebra. Work through the GeoGebra Geometry QuickStart Tutorial found in the Help section of the GeoGebra site.

4. From Drawing to Constructing

   Now that you have had a chance to play around, let's take a step back to reflect. The difference between constructing shapes on paper and constructing figures in a dynamic geometry environment is the difference between night and day. On paper, you can only offer a
finite number of examples and then ask that students extrapolate and imagine a generalized understanding of the relevant geometric properties. With dynamic geometry, you can grab and drag a corner or side of a shape and watch it transform through an infinite number of fluidly changing examples making the geometric constants powerfully evident. One can imagine, in ancient times, the great advancement in mathematics when someone made the leap from drawing in the sand to drawing on paper. The jump from paper to this dynamic geometry environment is really no less profound.

Within the dynamic geometry environment, we differentiate between drawing and constructing. As illustrated by Markus Hohenwarter’s squares which you encountered during the QuickStart Geometry Tutorial, shapes that appear to be squares may or may not pass the drag test. A shape that collapses when you grab and drag a corner or side is referred to as a drawing. One that maintains its integrity is referred to as a construction or as a figure. In dynamic geometry, the drag test is one place where geometry comes to life.

The art of constructing figures in a dynamic geometry environment requires a different way of thinking about geometry than the older process of constructing figures on paper with a straightedge and compass. A teacher-user of GeoGebra must learn the logic embedded in this new technology; a student-user learns the logic of GeoGebra at the same time as they are exploring the logic of geometry. The strength of the software is in how, largely, these logics are complementary. Of course, they are not exactly the same. In the abstract world of mathematics, a visualization is only an approximation of the abstract ideal, where lines are infinitely thin and
points have no dimensions. GeoGebra is a tool for visualizing mathematics. Just as with older tools like paper, pencil, straightedge and compass, one learns to use the tool by practice.

There are many different ways to construct a rectangle. The QuickStart tutorial offered one possible set of steps for constructing a rectangle. For an additional activity, check out this 3-minute Dynamic Rectangle Challenge video on Youtube.

While building in GeoGebra, starting from the first point, each new element can be placed either in relation to the existing elements or independently. These relationships matter. The curved arrow buttons in the top right of the GeoGebra window allow you to step backwards and forwards through your steps of construction. Many pre-constructed resources will also allow you to deconstruct what has been built by others using the optional Navigation Bar at the bottom of the window. One can also explore the Construction Protocol window, though this can be confusing for a new user.

If two lines cross where there is no point, you may want to mark that point using the Intersect tool in the dropdown menu hidden below the Point tool. Notice that this point of intersection won’t be independently dynamic (ie you won’t be able to drag it). It will move, however, as you drag one of the lines. Refer back to the example in the QuickStart Tutorial about how to construct an equilateral triangle. Devising constructions that transform elegantly through dragging is a challenge for beginners, and even for advanced users of GeoGebra.
Traditionally, geometry has been taught in high school through the study of deductive proofs. Problems with this traditional approach arise because many students remain unconvinced by these so-called proofs. Dynamic geometry allows for elegant visualizations of proofs. Alternately, dynamic geometry also allows for inductive proofs based on the logic of examining many examples until a student is convinced that something is true. After a novice student is convinced of this truth, then a formal deductive proof might have more meaning for them. Dynamic geometry offers a better fit for how many students actually think about geometry.

5. Envisioning a Dynamic Geometry Classroom

Now that you have played around with GeoGebra, try to imagine what introducing dynamic geometry software into your elementary classroom might look like. A first idealized image is likely of students engaged, either individually or in small groups, at the controls of a computer, or with an ipad-type mobile device in their hands, constructing geometric shapes and exploring their geometric properties and relationships. Realistically, for this picturesque moment to arrive, several preliminary steps need to be taken.

Imagine it like teaching a sport, such as basketball. You cannot simply give kids a ball in elementary school and expect that a basketball game will happen. There is a trajectory, a series of stages that need to be worked through. The game of basketball is learned experientially. It starts with getting familiar with the ball, how it bounces, how it rolls, how it feels to throw it and to catch it. A teacher needs to nurture in children a love of the ball and a sense of its importance, that one should value this object, not for its resale value, but because it is likely to be the center of action, the fulcrum about which all the games activity pivots. Playful repetition, scaffolded
intermediary games, and a skillful and knowledgeable coach are all necessary ingredients. Geometry, like basketball, is a specific cultural practice with a steep, but not insurmountable learning curve.

As a math coach in elementary school, what are the classroom activities needed to prepare young children for a successful mathematical future in upper school and beyond? What is the math equivalent of good hand-eye coordination and a love of the game of basketball, and how is that achieved? If you are approaching mathematics in this way, you have already made a significant step towards teaching geometry dynamically.

Thinking like a dynamic math coach, one can imagine several classroom configurations. A skillful teacher weaves between these different configurations, providing the needed support and guidance while allowing ample opportunities for free play and exploration, searching always for that sweet spot of challenge where each student is performing at the outer limit of their comfort zone and ability.

Three basic configurations are as follows:

- **Teacher-guided group explorations**: With dynamic geometry software, these likely involve a projector or a smartboard. However, these discussions could even involve off-line manipulatives, such as paper-folding or tangram puzzles, or perhaps a group search for shapes (either inside the classroom or outside in a field), or perhaps games. Different than demonstrations or lectures, this mode of teaching invites discussion, interaction, and open-ended exploration.

- **Individual or small group, student-led activities**: These pre-designed activities invite the student to work independently but in a structured, scaffolded way. With GeoGebra, this can be done using dynamic worksheets which allow a student to take the controls,
but with a limited set of options and some clear direction. Offline, these could be worksheets, puzzles, games, or other structured activities.

- **Independent work:** This is the idealized classroom described above, with students in control, constructively engaged in dynamic problem solving and increasingly complex mathematical modeling and geometric constructing. This state of working constructively and independently is the goal of teaching -- the un-modified game of basketball -- but at the elementary level, and depending on your particular students, it may only be achieved in fleeting moments and after considerable practice and preparation.

In the dynamic geometry classroom, student engagement is a measure of effectiveness. One can feel it and see it in the way children communicate, in their gestures and expressions. Like working with puzzles, it is key to be tuned in to the students' frustration levels, fostering a zone of productive struggle and even some frustration, but also providing support and guidance when needed, and modeling and nurturing a positive attitude towards learning.

6. **A Sample Lesson: Introducing Symmetry**

Follow this link to a GeoGebra Worksheet that shows a sample series of lessons suggesting [how a teacher might introduce Symmetry to their Grade 4 class](https://www.example.com/symmetrylesson).
7. GeoGebra’s Multiple Perspective Views

Opening GeoGebra in your web browser, and selecting Start GeoGebra to access the app through a web browser, one is immediately offered a choice of Perspectives. In GeoGebra, each perspective provides a view of the same mathematical information in a different system of notation. By viewing several perspectives side-by-side, one is afforded a powerful visualization of the dynamic relationships between geometry, algebra, and spreadsheet data.

Much of this is geared towards explorations at the middle and upper school levels. However, at an elementary level, it is worth thinking backwards about how we can distill some of this to make sense for young learners and to set the stage for their future explorations. A spreadsheet in its simplest form is just a two-column table.

The Algebra views in GeoGebra are divided into two levels of complexity. The CAS (Computer Algebra System) view deals with complex functions while the simpler Algebra view
offers a list of all the points, lines and polygons in the Geometry view, labeled with letters and coordinates. The simpler Algebra view also allows for basic formulas and calculations.

If you decide to explore GeoGebra through the downloadable app, it will by default open with a **Geometry view** section on the right and an **Algebra view** section on the left. The Geometry view will contain the X and Y Axes by default. This is also the layout that appears if you choose the Algebra view on the web start screen. **To get rid of the X-Y Axes**, right-click (or control-click on a Mac) inside the Geometry view section and de-check Axes on the pop-up menu.
You also have the option here to get rid of labels by right-clicking a point, a line, or any other element and de-checking Label.

You can add the Algebra view section by selecting the **Shapes icon** in the top right of the window and then the **3 Stacked Squares icon** and selecting **Algebra** from the dropdown menu.

**You can hide elements** in the **Geometry view** by deselecting the blue or grey button next to that element in the Algebra view, or, alternately, by right-clicking on the element in the Geometry view and de-checking **Show**.

To access a more comprehensive set of controls, click on the **Gear Wheel icon** to get to the **Settings Control Panel**.

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### 8. The GeoGebra Gap for Beginners

GeoGebra definitely requires an investment of time up front to learn how to use it and to develop the confidence and experience needed to lead classroom explorations and activities, to construct the needed dynamic worksheets, and to guide and support student-led construction projects. The ability to troubleshoot, especially, is developed through experience and mentorship. All aspects of this process can be made easier by a supportive environment and a network of more-experienced users.

Based on a quick surf through GeoGebraTube, one gets the impression that this software tool is more geared towards work at the upper levels. The discussion seems to start at an intermediate level, often with the split Algebra-Geometry view that shows the dynamic
relationship between an algebraic function and its corresponding graph. Returning to the metaphor of basketball, what GeoGebra currently lacks is an effective beginner-level program. Users of any age, including teachers, who cannot jump into this game with a degree of mathematical facility are left on the sidelines, or perhaps worse, they will find themselves in the midst of a complicated play without the skills or understanding needed to succeed.

The QuickStart Geometry tutorial provides a good orientation but it is clearly not enough. The GeoGebra tutorials page also offers an interactive book called Basic Geometry Constructions which goes into slightly more depth than the tutorial, and also a brief manual for GeoGebra 5.0, but there is no plan or set of guidelines that I have found for using GeoGebra at the elementary level. This gap currently makes it difficult for an inexperienced teacher to incorporate GeoGebra into their classroom.

I believe that GeoGebra has the potential to be a powerful tool for elementary-level teachers and students, despite the fact that the currently available supports are woefully underdeveloped. This gap can and should be closed.

In the next section, I offer a quick orientation for users wanting to share resources through GeoGebraTube. This is really the heart of the problem, or perhaps, more constructively, the frontier of this emerging network. Let's take a critical look at GeoGebraTube and then explore some strategies for how we can make better use of this valuable tool.
9. The Idiosyncrasies of GeoGebra’s Sharing Platform

Geogebra creations are called applets and can be saved. In the desktop app, these save as (.ggb) files, and can be emailed or reopened later for continued work.

One or more applets can be combined with text, images, video, and/or web-links to create a worksheet. Worksheets can only be created online, and are stored and shared via one’s free GeoGebraTube account. In a worksheet, an applet is typically presented without a tool pallet, or with just a few tools. Worksheet users are invited to manipulate the shared applet, but they cannot save the changes they make and the applet easily resets to its starting construction. Essentially, a worksheet limits a user’s access to the controls.

Up to this point, it all seems fairly logical. The system, however, (as of this writing in March, 2016) remains quite idiosyncratic and will require some getting used to.

If you are working with the web-based app, an applet can only be saved to your GeoGebraTube account and appears on your homepage alongside any other materials that you have created. If you want to reopen a saved applet to edit it and make changes, you have two options, neither of which are flawless.

In the first option, you click on the Pencil edit icon on the toolbar at the top right of the applet window. At this point, the applet instantly converts into a worksheet. This option is available only to owners of the applet, although a non-owner can create a new worksheet and import the applet themselves. In your own worksheet (if you are logged in) you can click the Pencil edit icon to enter edit mode. The process of editing an applet element in worksheet edit mode becomes much more cumbersome, however, than working in the regular web app.

To edit, again, click a second Pencil edit icon at the top right of the applet element and then click the Edit Applet button at the bottom right. When finished editing, click Done to exit.
the Edit Applet window, click **Done** again to exit the element, and then click the **Save and Close** button at the bottom of the worksheet. Miss a step and you could easily lose your work.

A few notes on editing in worksheet edit mode:

- If your applet pixel dimensions are set too large, it can be difficult to reach the **Done** button at the bottom right of the Edit Applet window. Reset the dimensions in the settings below the applet window, but be careful: The dimensions are live, so if you delete the number your app will disappear, and if you enter an interim number that is too large, you can overwhelm your computer.
• You can easily import other people’s applets into your own worksheets. GeoGebra automatically imports links to the original creator’s applet, as well as to the sequence of modified versions, at the bottom of the worksheet.

• You can import applets from one worksheet into another individually, even if the source worksheet contains multiple applets.

The second option for reopening an existing, saved applet online works either for one that you own or for an applet owned by someone else. Open the applet and click on the Share button at the top right of the window. On the Share screen, you are given an option to Open in GeoGebra Web App, which will take you to the web app with the applet construction at your disposal. In this case, the attribution metadata is lost and the resaved app may appear to have been created entirely by you.
GeoGebra resources also can take other forms such as books, tasks, tools, exercises, links, and spreadsheet data files. A book allows one to compile a series of worksheets. An understanding of the other types of resources, some of which remain ill-defined, is not essential for a beginner user.

10. Creating and Sharing Quality Worksheets

Despite the idiosyncrasies, creating applets and worksheets in GeoGebra is not at all impossible and is, in fact, quite worth the effort. As described in the sample lesson earlier, pre-constructed applets offer a key mechanism for creating structure during teacher-led discussions and for modeling concepts dynamically and interactively. The right construction can be the basis for powerful group explorations and the right worksheet can provide a student with a degree of scaffolded independence before they are ready to begin making constructions on their own.

At present, you won’t find many resources on GeoGebraTube that have been shared with either of these scenarios in mind. If you are a teacher creating new resources to use and if you decide to share your creations with the community, here are a few tips to help make your shared applets and worksheets more user-friendly.

- Remember to manage your sharing settings. Resources are publicly shared unless you change the default settings either at the time of first saving, or later through the Worksheet Settings at the bottom of the worksheet. There is no need to publicly share an unfinished applet, which will only serve to clutter the community sharing system.
● Provide a text note. If it is a resource for teachers, offer suggestions about how and in what context you intend it to be used, and consider suggesting good questions for class discussions. Directions should be minimal, but are sometimes necessary.

● In general, as much as possible, try to show what you have done. Share your construction steps and provide useful explanations.

● If a worksheet is for students, try to make it interactive and provide opportunities for meaningful, scaffolded construction and exploration. Don’t just provide demonstrations. The real learning and deeper engagement involve active participation from students.

● As a default, GeoGebra will hide the tools on a worksheet, so that student-users don’t get distracted. Sometimes though, it makes sense to make one or more tools available for worksheet users. Consider how best to maximize students’ engagement.

● As a default, In the worksheet edit mode, the Enable Pan & Zoom option in the Advanced settings below the applet window is enabled. Consider disabling it to give the applet more stability for a user, especially one who is using a mouse with a scroll wheel.
11. Searching, Commenting and Folksonomy Tagging

Searching:

Sorting through the 370,000 shared resources on GeoGebraTube is difficult. GeoGebraTube offers a few featured resources, as well as the most recently added resources. When you search, you can sort resources by Type, Age and Language.

Users seem to use different strategies for sharing resources. Under Type, you can filter to worksheets that just contain Links to external websites where users have posted resources which are sometimes of a higher caliber. As well, the Tools filter is intriguing.

In the bottom righthand corner of the Search screen, you also have additional options to sort by Relevance, Date, Title, or Rating. Ultimately, none of these options are very satisfying or helpful.
Commenting:

Applets and Worksheets do allow for Comments, but this facility is buried and therefore underutilized. You can find the Comment section by clicking on the 3 Stacked Squares icon on the top right of the applet or worksheet window and choosing About.
Folksonomy Tagging:

The metadata mechanism within GeoGebraTube with the greatest potential, in the long run, for helping elementary teachers sort through the sea of resources in search of supportive materials is the **keyword tagging system**. This system is proactively encouraged by GeoGebraTube with a pop-up window each time you save. You can also add tags to other people’s resources that you view by going to the **About** screen.

Some thought needs to be given to how to most effectively tag resources, considering that every region follows a different curriculum standard. Here, below are a few useful tags that you could consider using:

- **#Elementary** -- for any resources an elementary teacher might find useful
- **#BCED4** -- indicating a fit with British Columbia Curriculum Standards grade level
- **#CCSS6** -- indicating a fit with for Common Core grade level
- **#Symmetry** -- by topic, #Squares, #Triangles, #Tessellations
- **#Teacher** -- for resources to be used as teacher-led activities
- **#StudentWork** -- for materials being shared by students
- **#Worksheet** -- intended for use by students independently
- **#Simulation**
- **#Proof**
- **#Construction**
- **#Challenge**
- **#Puzzle**
- **#Offline** -- for directions for dynamic-inspired offline materials and manipulatives

Perhaps the use of hashtags (#) could indicate a more purposeful use of tagging and contribute to a cooperative effort among elementary school teachers.
12. Feedback & Further Discussion

You can provide feedback, comments, or ask questions about this tutorial directly on GeoGebraTube under the Symmetry Lesson Plan Worksheet which also provides a link back to this pdf file. Just click on the 3 Stacked Squares icon in the top right toolbar and choose About to access the Comments page.

You can also join the discussion in the GeoGebra Forum under the Ideas section under the topic heading: Thinking Creatively about Teaching Geometry in Elementary Tutorial.

13. About Me

My name is Meril Rasmussen and I am a graduate student at the University of British Columbia in the Masters of Educational Technology program, which I am doing online from Rio de Janeiro where I live with my partner and our two children, who are 6 and 9. My undergraduate degree is in Interdisciplinary Fine Art from the Nova Scotia College of Art and Design (NSCAD, class of ‘99) and I have an honour’s degree from the AFDA film school in Johannesburg, South Africa (2005). I have worked as community organizer and as a documentary filmmaker in Canada, South Africa, India, and now Brazil.

In India between 2009 and 2012, I made a film about a group of elementary-level math educators who, from their base in West Delhi, run a model school while producing math
manipulatives in two factories and training teachers throughout Northern India to use them. The group is called *Jodo Gyan* and one of the founders, Usha Menon, has been deeply influenced by the Realistic Math Movement in the Netherlands through her direct contact with the Freudenthal Institute. This film, and the process of engagement with Jodo Gyan that it involved, re-awakened in me a long-neglected love of mathematics and education.

My father was a high school math teacher, my grandmother a celebrated leader of the New Math Movement in 1960’s Philadelphia. My uncles founded Key Curriculum Press in California in 1971 and 20 years later released the *Geometer’s Sketchpad* software. I’ve been watching this story unfold my whole life.

I am not, however, an experienced classroom teacher. I have ideas and opinions grounded in educational research, personal observations, and diverse life experiences. My personal strengths lie in the building of communities, in sharing and developing creative ideas, and increasingly in the design and development of virtual learning environments.
References


